

Hadrons with two heavy quarks

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Abstract

We review the spectroscopy and some properties of hadrons containing two charmed quarks, or more generally, two heavy quarks. This includes heavy baryons such as (bcu) , and possible exotic multi-quark states.

1 Introduction

Baryons with two heavy quarks and one light quark, hereafter denoted (QQq) , intimately combine two extreme regimes of hadron structure. There is first the slow relative motion of the two heavy quarks, very similar to the quark–antiquark motion in charmonium and bottomonium. In both cases, the heavy constituents experience an adiabatic potential generated by the light degrees of freedom. The second aspect of (QQq) is the relativistic motion of the light quark q , which is presumably very similar for (ccq) , (bcq) , and (bbq) , providing another example of heavy quark symmetry.

A rich spectrum is expected. There are excitations of the relative motion of the two heavy quarks in the lowest Born–Oppenheimer potential. One can also get excitations of the light quark, or a combined excitation of both degrees of freedom.

The ground state of each flavour configuration cannot do anything but decay weakly, by disintegration of one of the heavy quarks, and sometimes by exchange of a W -boson between the constituents. A variety of final states are accessible, with no, some, or more Cabibbo suppression. We have here an ideal laboratory for studying weak interactions and subsequent hadronisation.

If (QQq) spectroscopy becomes accessible to experiment, it will also be possible to look at exotic mesons with two heavy quarks, $(QQ\bar{q}\bar{q})$. They have been predicted to be stable on the basis of the flavour independence of the static interquark potential. Other approaches have led to similar conclusions. Current models give stability for ratios (M/m) of quark masses corresponding to $(bb\bar{q}\bar{q})$ or higher. However, reasonable long-range forces might well push down this ratio, so that some $(cc\bar{q}\bar{q})$ could become serious candidates to stability.

In this review, I shall briefly summarize these aspects of double heavy-flavour spectroscopy. General references are [1, 2, 3, 4] for (QQq) spectroscopy in potential models, [4, 5] for decays of these (QQq) , [6, 7] for $(QQ\bar{q}\bar{q})$ exotics in simple models, while a comparison with atomic physics is attempted in [8], and another approach is discussed in [9, 10]. It is hoped that this Workshop will stimulate further investigations.

2 Relations among ground state masses

The value of a peculiar (QQq) mass is interesting only when compared with that of other flavour configurations. In the past, regularities have been noticed in the baryon spectrum, such as the Gell-Man–Okubo mass formula, or the equal-spacing rule of the decuplet. One possible interpretation in the modern language is based on *flavour independence*. The binding potential is the same whatever quark experiences it. This property is a consequence of the gluons being coupled to the colour rather than to the isospin, or hypercharge, or mass of the quarks, at least before any relativistic correction is written down. We shall come back on flavour independence in Sec. 5, and stress the analogy with atomic physics, where the same $-1/r$ potential binds positronium, hydrogen and protonium atoms.

In the meson sector, we expect the lowest ($b\bar{c}$) meson approximately half between J/Ψ and Υ . In a flavour-independent potential, this is in fact a lower bound [11], i.e., we have

$$2(b\bar{c}) \geq (c\bar{c}) + (b\bar{b}). \quad (1)$$

If one knows the excitation spectrum of ($c\bar{c}$) and ($b\bar{b}$), one can extract model-independent bounds on the average kinetic energy in the ground state, which governs the evolution of the ground-state energy when the reduced mass varies. This leads to an upper bound on the lowest ($b\bar{c}$) state [4], and all predictions of realistic potentials nicely cluster near $6.26 \text{ GeV}/c^2$ [12] in between the lower and the upper bounds provided by flavour independence.

Similar regularity patterns are expected in the baryon sector (the mathematics of the 3-body problem is of course more delicate than that of the 2-body one, and sometimes requires some mild conditions on the shape of the confining potential, which are satisfied by all current models [3]). For instance, one expects an analogue of (1)

$$2(ccq) \geq (ccq) + (qqq) \quad (2)$$

which leads to an upper bound $(ccq) \leq 3.7 \text{ GeV}$ for the c.o.m. of the ground-state multiplet of (ccq). An upper bound can also be derived for (ccs). On the other hand, the convexity relation

$$2(bcq) \geq (ccq) + (bbq), \quad (3)$$

cannot be tested immediately, as well as the even more exotic-looking [13]

$$3(bcq) \geq (bbb) + (ccc) + (qqq), \quad (4)$$

and its analogue with $q \rightarrow s$. Of more immediate use is the relation

$$(bcq) \geq (bqq) + (ccq) - (qqq), \quad (5)$$

which leads to a rough lower bound $(bcq) \geq 6.9 \text{ GeV}/c^2$, if one inputs the following rounded and spin-averaged values: $(bqq) = 5.6$, $(ccq) = 2.4$, and $(qqq) = 1.1 \text{ GeV}/c^2$.

To derive these inequalities, one uses the Schrödinger equation, even for the light quarks. Very likely, the regularities exhibited by flavour-independent potentials also hold in more rigorous QCD calculations and in the experimental spectrum. Any failure of the above inequalities would be very intriguing.

Sometimes, one can be more precise, and derive inequalities that include spin–spin corrections, for instance relations between $J^P = (1/2)^+$ baryons with different flavour content. See [3] for details.

Another mathematical game triggered by potential models consists of writing inequalities among meson and baryon masses. The basic relation is [3]

$$2(q_1 q_2 q_3) \geq (q_1 \bar{q}_2) + (q_2 \bar{q}_3) + (q_3 \bar{q}_1), \quad (6)$$

obtained by assuming that the potential energy operators fulfill the following inequality

$$2V_{qqq}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \geq \sum_{i < j} V_{q\bar{q}}(|\mathbf{r}_i - \mathbf{r}_j|), \quad (7)$$

which holds (with equality) for a colour-octet exchange, in particular one-gluon exchange, and for the simple model

$$V_{q\bar{q}}(r) = \lambda r, \quad V_{qqq} = \lambda \min_J (d_1 + d_2 + d_3) \quad (8)$$

where d_i is the distance from the i -th quark to a junction J whose location is adjusted to minimize V_{qqq} [14]. We obtain for instance [1] $(ccq) \geq 3.45 \text{ GeV}/c^2$ for the $(1/2)^+$ state. This is rather crude, not surprisingly. Years ago, Hall and Post [15] pointed out in a different context that the pairs are not at rest in a 3-body bound state, and that their collective kinetic energy is neglected in inequalities of type (6).

3 Spectrum of doubly flavoured baryons

Computing the (QQq) energies in a given potential model does not raise any particular difficulty. The 3-body problem is routinely solved by means of the Faddeev equations or variational methods. On the other hand, successful approximations often shed some light on the dynamics. In particular, the Born–Oppenheimer method works very well for large ratios (M/m) of the quark masses. At fixed QQ separation R , one solves the 2-centre problem for the light quark q . The energy of q is added to the direct QQ interaction to generate the effective potential $V_{QQ}(R)$ in which the heavy quarks evolve. One then computes the QQ energy and wave function. Note that one can remove the centre-of-mass motion exactly, and also estimate the hyperfine corrections.

The physics behind the Born–Oppenheimer approximation is rather simple. As the heavy quarks move slowly, the light degrees of freedom readjust themselves to their lowest configuration (or stay in the same n -th excitation, more generally). At this point, there is no basic difference with quarkonium. The $Q\bar{Q}$ potential does not represent an elementary process. It can be viewed as the effective interaction generated by the gluon field being in its ground-state, for a given $Q\bar{Q}$ separation.

The results shown in Table 1 come from the simple potential

$$V = \frac{1}{2} \sum_{i < j} \left[A + B r_{ij}^\beta + \frac{C}{m_i m_j} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \delta^{(3)}(\mathbf{r}_{ij}) \right], \quad (9)$$

with parameters $\beta = 0.1$, $A = -8.337$, $B = 6.9923$, $C = 2.572$, in units of appropriate powers of GeV. The quark masses are $m_q = 0.300$, $m_s = 0.600$, $m_c = 1.905$ and $m_b = 5.290 \text{ GeV}$. The $1/2$ factor is a pure convention, although reminiscent from the discussion of inequalities (6) and (7). The smooth central term can be seen as a handy interpolation between the short-range Coulomb regime modified by asymptotic-freedom corrections and an elusive linear regime screened by pair-creation effects. The spin-spin term is treated at first order to estimate M_0 . This model fits all known ground-state baryons with at most one heavy quarks.

Table 1: Masses, in GeV, of (QQq) baryons in a simple potential model. We show the spin-averaged mass \bar{M} , and the mass M_0 of the lowest state with $J^P = (1/2)^+$.

State	ccq	ccs	bcq	bcs	bbq	bbs
\bar{M}	3.70	3.80	6.99	7.07	10.24	10.30
M_0	3.63	3.72	6.93	7.00	10.21	10.27

A more conventional Coulomb-plus-linear potential was used in Ref. [1], with similar results. One remains, however, far from the large number of models available for $(b\bar{c})$ [12], and the non-relativistic treatment of the light quark might induce systematic errors. The uncertainty is then conservatively estimated to be $\pm 50 \text{ MeV}$, as compared to $\pm 20 \text{ MeV}$ for $(b\bar{c})$. Note also that the b -quark mass m_b is tuned to reproduce the experimental mass of Λ_b at $5.290 \text{ GeV}/c^2$, and this latter value is not firmly established.

The Born–Oppenheimer framework leaves room for improvements. A relativistic treatment of the light quark was attempted in [1], using the bag model. For any given QQ separation, a bag is constructed in which

the light quark moves. The shape of the bag is adjusted to minimize the energy. In practice, a spherical approximation is used, so that the radius is the only varying quantity. The energy of the bag and light quark is interpreted as the effective QQ potential. Unlike the rigid MIT cavity, we have a self-adjusting bag, which follows the QQ motion. Again, this is very similar to the bag model picture of charmonium [16].

Unfortunately, there are variants in the bag model, with different values of the parameters, and with or without corrections for the centre-of-mass motion. These variants lead to rather different values for the (ccq) masses [1]. This contrasts with the clustered shoots of potentials models, and deprives the bag model of predictive power in this sector of hadron spectroscopy.

It is hoped that the QQ potential will be calculated by lattice or sum-rule methods.

The excitation spectrum of (QQq) baryons has never been calculated in great detail, at least to our knowledge. In Ref. [1], an estimate is provided for the spin excitation (ground state with $J^P = (3/2)^+$), the lowest negative-parity level, and the radial excitation of the ground state.

The spin excitation is typically 100 MeV above the ground state, and thus should decay radiatively, with a $M1$ transition. The orbital and radial excitations of (ccq) are unstable, since they can emit a pion. The radial excitation of (ccs) can decay into $(ccq) + K$, but the orbital excitation cannot, and thus should be rather narrow, since restricted to $(ccs) + \gamma$, or to the isospin-violating $(ccs) + \pi^0$.

4 Decay of heavy baryons

The ground state of (QQq) decays weakly, with a great variety of final states. For instance, the remaining heavy flavour can stay in the baryon, or join the meson sector. Moreover, we have Cabibbo allowed, suppressed, or doubly suppressed modes. We refer to Savage et al. [5] for a comprehensive survey of 2-body channels of interest.

Inclusive decay rates are also of great importance. The difference between the D^0 and D^+ lifetimes tells us that the charmed quark, while decaying, does not ignore its environment. The main process is $c \rightarrow s + W$, and $W \rightarrow u\bar{d}$ for hadronic modes, but one should also consider W -exchange contribution for D^0 , interferences between the two \bar{d} in D^+ decay, $c\bar{s}$ annihilation for D_s , etc.

The lifetimes of single-charm baryons have been analysed by Guberina et al. [17]. The annihilation diagram requires antiquarks from the sea, and presumably does not play a very important role. On the other hand, W -exchange does not suffer from helicity suppression. We also have two types of interferences: between constituent u and u from W decay, and between constituent s and s from c transmutation. The predictions of [17]

$$\tau(\Omega_c^0) \lesssim \tau(\Xi_c^0) < \tau(\Lambda_c^+) < \tau(\Xi_c^+), \quad (10)$$

seems confirmed by recent data. If one extrapolates their analysis toward the (ccq) sector, one predicts [1]

$$\tau(\Xi_{cc}^+) < \tau(\Omega_{cc}^+) < \tau(\Xi_{cc}^{++}). \quad (11)$$

5 Exotic mesons with two heavy quarks

The situation and the perspectives for the pentaquark will be reviewed by Moinester [18]. The pentaquark is an exotic baryon ($B = 1$) with charm (or heavy flavour) $C = -1$, i.e., a $(\bar{Q}qqqq)$ structure. We shall discuss another possible multiquark, the tetraquark, with $B = 0$ and $C = 2$. The main difference, besides these quantum numbers, is that the pentaquark is tentatively bound by chromomagnetic forces, while the tetraquark uses a combination of flavour-independent chromoelectric forces, and Yukawa-type of long range forces.

Recently, Törnqvist [9], and Manohar and Wise [10] studied pion-exchange between heavy mesons, and stressed that, among others, some DD^* and BB^* configurations experience attractive long-range forces. By itself, this Yukawa potential seems unlikely to bind DD^* , but might succeed for the heavier BB^* system.

Years ago, Ader et al. [6] showed that $(QQ\bar{q}\bar{q})$ should become stable for very large quark-mass ration (M/m) , a consequence of the flavour independence of chromoelectric forces. The conclusion was confirmed in subsequent studies [7].

In the limit of large (M/m) , $(QQ\bar{q}\bar{q})$ bound states exhibit a simple structure. There is a localized QQ diquark with colour $\bar{3}$, and this diquark forms a colour singlet together with the two \bar{q} , as in every flavoured antibaryon. In other words, this multiquark uses well-experienced colour coupling, unlike speculative mock-baryonia or other states proposed in “colour chemistry” [19], which contain clusters with colour 6 or 8.

The stability of $(QQ\bar{q}\bar{q})$ in flavour-independent potentials is analogous to that of the hydrogen molecule [8]. If one measures the binding in units of the threshold energy, i.e., the energy of two atoms, one notices that the positronium molecule $(e^+e^+e^-e^-)$ with equal masses is bound by only 3%, while the very asymmetric hydrogen reaches 17%. This can be understood by writting the molecular Hamiltonian as

$$\begin{aligned} H &= H_S + H_A \\ &= \left(\frac{1}{4M} + \frac{1}{4m} \right) (\mathbf{p}_1^2 + \mathbf{p}_2^2 + \mathbf{p}_3^2 + \mathbf{p}_4^2) + V \\ &\quad + \left(\frac{1}{4M} - \frac{1}{4m} \right) (\mathbf{p}_1^2 + \mathbf{p}_2^2 - \mathbf{p}_3^2 - \mathbf{p}_4^2) \end{aligned} \quad (12)$$

The Hamiltonian H_S , which is symmetric under charge conjugation, has the same threshold as H , since only the inverse reduced mass $(M^{-1} + m^{-1})$ enters the energy of the (M^+m^-) atoms. Since H_S is nothing but a rescaled version of the Hamiltonian of the positronium molecule, it gives 3% binding below the threshold. Then the antisymmetric part H_A lowers the ground-state energy of H , a simple consequence of the variational principle.

In simple quark models without spin forces, we have a similar situation. The equal mass case is found unbound, and $(QQ\bar{q}\bar{q})$ becomes stable, and more and more stable, as (M/m) increases. One typically needs $(bb\bar{q}\bar{q})$, with $q = u$ or d , to achieve binding with the nice diquark clustering we mentioned. However, if one combines this quark attraction with the long-range Yukawa forces, one presumably gets binding for $(bb\bar{q}\bar{q})$ with DD^* quantum numbers. A more detailed study is presently under way [20].

The experimental signature of tetraquark heavily depends on its exact mass. Above DD^* , we have a resonance, seen as a peak in the DD^* mass spectrum. Below DD^* , one should look at $DD\gamma$ decay of tetraquark. If it lies below DD , then it is stable, and decays via weak interactions, with a lifetime comparable to that of other charmed particles.

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